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**HYBRID CONTROL SYSTEMS: DESIGN AND ANALYSIS FOR AEROSPACE  
APPLICATIONS**

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## Summary

The objective of this research was to contribute to the fundamental understanding of hybrid control systems and to explore the use of hybrid feedback in problems of interest to the Air Force. It is our contention that hybrid control's vast potential has barely been tapped and a primary reason for this is that the theory governing hybrid systems has remained mostly "ad-hoc" in nature. We aimed to provide a solid, foundational understanding of hybrid systems that will enable the vast potential of hybrid control to be realized. On the analysis side, we investigated the implications of different hybrid semantics, the simulation of hybrid systems, and the interconnection of hybrid systems, all of which are important for understanding hybrid control design and evaluation. On the control side, we focused on synthesis of robust hybrid control systems. Special attention was given to problems involving online decision making, especially for unmanned air vehicles, on time scales that are similar to the natural time scales of the physical system for which decisions are being made. We also investigated the use of hybrid control in traditional feedback loops, including the use of reset control systems, hybrid logic in anti-windup control, and the design of hybrid patchy vector fields.

## Research Publications

The research supported by this grant resulted in 30 published journal papers, 44 refereed conference papers, and 7 book chapters. These publications are listed below.

### Journal papers

1. "Lyapunov conditions for input-to-state stability of impulsive systems", Hespanha JP, Liberzon D, Teel AR, *Automatica*, vol.44, no.11, Nov. 2008, pp. 2735-44.
2. "Stability properties of reset systems", Nesic D, Zaccarian L, Teel AR, *Automatica*, vol.44, no.8, Aug. 2008, pp. 2019-26.
3. "Invariance principles for switching systems via hybrid systems techniques", Goebel R, Sanfelice RG, Teel AR, *Sys & Control Letters*, vol.57, no.12, Dec. 2008, pp. 980-6.
4. "Smooth Lyapunov functions for hybrid systems Part II: (Pre)asymptotically stable compact sets", Chaohong Cai, Goebel R, Teel AR, *IEEE Transactions on Automatic Control*, vol.53, no.3, April 2008, pp. 734-48.
5. "Exponential stability of impulsive systems with application to uncertain sampled-data systems", Naghshtabrizi P, Hespanha JP, Teel AR, *Systems & Control Letters*, vol.57, no.5, May 2008, pp. 378-85.
6. "Input-output-to-state stability for discrete-time systems", Chaohong Cai, Teel AR, *Automatica*, vol.44, no.2, Feb. 2008, pp. 326-36.
7. "The 12 anti-windup problem for discrete-time linear systems: definition and solutions", Grimm G, Teel AR, Zaccarian L, *Systems & Control Letters*, vol.57, no.4, April 2008, pp. 356-64.
8. "Anti-windup synthesis for linear control systems with input saturation: achieving regional, nonlinear performance", Tingshu Hu, Teel AR, Zaccarian L., *Automatica*, vol.44, no.2, Feb. 2008, pp. 512-19.
9. "A magnitude and rate saturation model and its use in the solution of a static anti-windup problem", Galeani S, Onori S, Teel AR, Zaccarian L, *Systems & Control Letters*, vol.57, no.1, Jan. 2008, pp. 1-9.
10. "Invariance principles for hybrid systems with connections to detectability and asymptotic stability", R.G. Sanfelice, R. Goebel, and A.R. Teel, *IEEE Transactions on Automatic Control*, vol. 52, no. 12, Dec. 2007, pp. 2282-97.



11. "Hybrid feedback control and robust stabilization of nonlinear systems", Prieur C, Goebel R, Teel AR. *IEEE Trans. Auto. Control*, vol.52, no.11, Nov. 2007, pp. 2103-17.
12. "Nominally robust model predictive control with state constraints", Grimm G, Messina MJ, Tuna SE, Teel AR. *IEEE Trans. Auto. Cont.*, vol.52, no.10, Oct. 2007, pp. 1856-70.
13. "Stability of wireless and wireline networked control systems", Tabbara M, Nesic D, Teel AR. *IEEE Trans. on Automatic Control*, vol.52, no.9, Sept. 2007, pp. 1615-30.
14. "Smooth Lyapunov functions for hybrid systems – Part I: Existence is equivalent to robustness", C. Cai, A.R. Teel, and R. Goebel, *IEEE Transactions on Automatic Control*, vol. 52, no. 7, July 2007, pp. 1264-1277.
15. "Sufficient conditions for robustness of KL-stability for difference inclusions", C.M. Kellett and A.R. Teel, *Mathematics of Control, Signals, and Systems*, Springer-Verlag, Vol.19, no. 3, pp. 183-205, August 2007.
16. "Asymptotic characterizations of input-output-to-state stability for discrete-time systems", C. Cai and A.R. Teel, *Sys. & Cont. Let.*, vol. 56, no. 6, June 2007, pp. 408-415.
17. "Characterization of forced vibration for difference inclusions: a Lyapunov approach", T. Hu and A.R. Teel, *IEEE Trans. Cir. and Sys. I: Reg. papers*, v. 54, no. 6, June 2007.
18. "A Lyapunov proof of an improved maximum allowable transfer interval for networked control systems", D. Carnevale, A.R. Teel and D. Nesic, *IEEE Transactions on Automatic Control*, vol. 52, no. 5, May 2007, pp. 892-7.
19. "Constructive nonlinear anti-windup design for exponentially unstable linear plants", L. Zaccarian, S. Galeani and A.R. Teel, *Sys. & Cont. Let.*, 56 (5), May 2007, pp. 357-65.
20. "Anti-windup for marginally stable plants and its application to open water channel control problems", L. Zaccarian, L. Yuping, E. Weyer, M. Cantoni, and A.R. Teel, *Control Engineering Practice*, vol. 15, no. 2, Feb. 2007, 261-72.
21. "On 'uniformity' in definitions of global asymptotic stability for time-varying nonlinear systems", A.R. Teel and L. Zaccarian, *Automatica*, vol. 42, no. 12, Dec. 2006, pp. 2219-2222.
22. "Smooth time-varying stabilization of driftless systems over communication channels", A.R. Teel, A. Loria, E. Panteley, and D. Popovic, *Systems & Control Letters*, vol. 55, no. 12, Dec. 2006, pp. 982-991.

23. "Stability and performance for saturated systems via quadratic and nonquadratic Lyapunov functions", T. Hu, A.R. Teel and L. Zaccarian, *IEEE Transactions on Automatic Control*, vol. 51, no. 11, Nov. 2006, pp. 1770-1786.
24. "Stabilization of sampled-data nonlinear systems via backstepping on their Euler approximate model", D. Nesic and A.R. Teel, *Automatica*, vol. 42, no. 10, Oct. 2006, pp. 1801-1808.
25. "On the robustness of KL-stability for difference inclusions: smooth discrete-time Lyapunov functions", C.M. Kellett and A.R. Teel, *SIAM Journal on Control and Optimization*, v 44, n 3, 2006, p 777-800.
26. "Extremum seeking methods for optimization of variable cam timing engine operation" Popovic, D.; Jankovic, M.; Magner, S.; Teel, A. R. *IEEE Transactions on Control Systems Technology*, v 14, n 3, May, 2006, p 398-407.
27. "Conjugate convex Lyapunov functions for dual linear differential inclusions" Goebel, R.; Teel, A.R.; Hu, T.; Lin, Z. *IEEE Transactions on Automatic Control*, v 51, n. 4, April, 2006, p 661-666.
28. "Solutions to hybrid inclusions via set and graphical convergence with stability theory applications", Goebel, R.; Teel, A.R.; *Automatica*, v 42, n 4, April, 2006, p 573-587.
29. "Reduced order linear anti-windup augmentation for stable linear systems" Galeani, S.; Massimetti, M.; Teel, A.R.; Zaccarian, L.; *International Journal of Systems Science*, v 37, n 2, Feb 10, 2006, p 115-127.
30. "An anti-windup strategy for active vibration isolation systems", Teel, A.R.; Zaccarian, L.; Marcinkowski, J.; *Control Engineering Practice*, v 14, n 1, January, 2006, p 17-27.

#### Conference papers

31. "Analysis of hybrid systems resulting from relay-type hysteresis and saturation: A Lyapunov approach," Dai, Dan; Sanfelice, Ricardo G.; Hu, Tingshu; Teel, Andrew R., *47<sup>th</sup> IEEE Conference on Decision and Control*, pp. 2764-2769, Dec. 2008.
32. "Supervising a family of hybrid controllers for robust global asymptotic stabilization", Sanfelice, Ricardo G.; Teel, Andrew R.; Goebel, Rafal, *47<sup>th</sup> IEEE Conference on Decision and Control*, pp. 4700 – 4705, Dec. 2008.
33. "A Lyapunov-based small-gain theorem for hybrid ISS systems", Nesic, D.; Teel, A.R.; *47<sup>th</sup> IEEE Conference on Decision and Control*, pp. 3380 – 3385, Dec. 2008.



34. "Zeno behavior in homogeneous hybrid systems", Goebel, Rafal; Teel, Andrew R.; *47<sup>th</sup> IEEE Conference on Decision and Control*, pp. 2758 – 2763, Dec. 2008.
35. "Trends in nonlinear control", Astolfi, Alessandro; Nesic, Dragan; Teel, Andrew R.; *47<sup>th</sup> IEEE Conference on Decision and Control*, pp. 1870 – 1882, Dec. 2008.
36. "Robust hybrid source-seeking algorithms based on directional derivatives and their approximations", Mayhew, Christopher G.; Sanfelice, Ricardo G.; Teel, Andrew R.; *47<sup>th</sup> IEEE Conference on Decision and Control*, pp. 1735 – 1740, Dec. 2008.
37. "Lyapunov characterization of Zeno behavior in hybrid systems", Goebel, R., Teel, A.R.; *47<sup>th</sup> IEEE Conference on Decision and Control*, pp. 2752 – 2757, Dec. 2008.
38. "Hybrid control strategy for robust global swing-up of the pendubot", R. O'Flaherty, R. G. Sanfelice, and A.R. Teel, *Proceedings of the American Control Conference*, pp. 1424 – 1429, June 2008.
39. "Robust source-seeking hybrid controllers for nonholonomic vehicles", C.G. Mayhew, R.G. Sanfelice, and A.R. Teel, *Proceedings of the American Control Conference*, pp. 2722 - 2727, June 2008.
40. "A nested Matrosov theorem for hybrid systems", R.G. Sanfelice and A.R. Teel, *Proceedings of the American Control Conference*, pp. 2915 - 2920, June 2008.
41. "On robust, global stabilization of the attitude of an underactuated rigid body using hybrid feedback", A.R. Teel and R.G. Sanfelice, *Proceedings of the American Control Conference*, pp. 2909 - 2914, June 2008.
42. "On necessary and sufficient conditions for exponential and L2 stability of planar reset systems", D. Nesic, A.R. Teel and L. Zaccarian, *Proceedings of the American Control Conference*, pp. 4140 - 4145, June 2008.
43. "Uniting a high performance, local controller with a global controller: the output feedback case for linear systems with input saturation", C. Prieur and A.R. Teel, *Proceedings of the American Control Conference*, pp. 2903 - 2908, June 2008.
44. "IISS gain of dissipative systems", B. Jayawardhana, A.R. Teel, and E.P. Ryan, *46<sup>th</sup> IEEE Conference on Decision and Control*, Dec. 2007, pp. 3835-3840.
45. "A hybrid systems approach to trajectory tracking control for juggling systems" R.G. Sanfelice, A.R. Teel, R. Sepulchre, *46<sup>th</sup> IEEE Conf. on Decision and Control*, Dec. 2007, pp. 5282—5287.
46. "Hybrid systems techniques for convergence of solutions to switching systems" R. Goebel, R.G. Sanfelice, A.R. Teel, *46<sup>th</sup> IEEE Conf. on Decision and Control*, Dec. 2007, pp. 92--96.

47. "Robust source-seeking hybrid controllers for autonomous vehicles", C.G. Mayhew, R.G. Sanfelice, and A.R. Teel, *Proceedings of the American Control Conference*, New York City, NY, USA, July 2007, pp. 1185--1190.
48. "A hybrid control strategy for robust contact detection and force regulation", R. Carloni, R.G. Sanfelice, A.R. Teel and C. Melchiorri, *Proceedings of the American Control Conference*, New York City, NY, USA, July 2007, pp. 1461 – 1466.
49. "Results on existence of smooth Lyapunov functions for (pre-)asymptotically stable hybrid systems with non-open basins of attraction", C. Cai, A.R. Teel, and R. Goebel, *Proc. of the American Control Conf.*, New York City, NY, USA, July 2007, 3456 – 3461.
50. "Subtleties in robust stability of discrete-time piecewise affine systems", M. Lazar, M. Heemels, and A.R. Teel, *Proceedings of the American Control Conference*, New York City, NY, USA, July 2007, 3464 – 3469.
51. "A 'throw-and-catch' hybrid control strategy for robust global stability of nonlinear systems", R.G. Sanfelice and A.R. Teel, *Proceedings of the American Control Conference*, New York City, NY, USA, July 2007, pp. 3470 – 3475.
52. "Path-following in the presence of unstable zero dynamics: an averaging solution for nonlinear systems", D. Dacic, D. Nesic, and A.R. Teel, *Proceedings of the American Control Conference*, New York City, NY, USA, July 2007, pp. 4500 – 4505.
53. "Stability of delay impulsive systems with application to networked control systems", P. Naghshtabrizi, J.P. Hespanha, and A.R. Teel, *Proceedings of the American Control Conference*, New York City, NY, USA, July 2007, pp. 4899--4904.
54. "Set-point stabilization of SISO linear systems using first order reset elements", L. Zaccarian, D. Nesic, and A.R. Teel, *Proceedings of the American Control Conference*, New York City, NY, USA, July 2007, pp. 5808 – 5809.
55. "Analysis of systems with saturation/deadzone via piecewise-quadratic Lyapunov functions", D. Dai, T. Hu, A.R. Teel, and L. Zaccarian, *Proceedings of the American Control Conference*, New York City, NY, USA, July 2007, pp. 5822 – 5827.
56. "Further results on stability of networked control systems: a Lyapunov approach", D. Carnevale, A.R. Teel, D. Nesic, *Proceedings of the American Control Conference*, New York City, NY, USA, July 2007, pp. 1741-1746.
57. "A trajectory based approach for robust stability properties of infinite-dimensional systems", Y. Tan, D. Nesic, and A.R. Teel, *Proceedings of the 7<sup>th</sup> IFAC Symposium on Nonlinear Control Systems*, August 2007, Pretoria, South Africa.

58. "Relaxed characterizations of smooth patchy control Lyapunov functions", R. Goebel, C. Prieur, and A.R. Teel, *Proceedings of the 7<sup>th</sup> IFAC Symposium on Nonlinear Control Systems*, August 2007, Pretoria, South Africa.
59. "Uniting local and global output feedback controllers", C. Prieur and A.R. Teel, *Proceedings of the 7<sup>th</sup> IFAC Symposium on Nonlinear Control Systems*, August 2007, Pretoria, South Africa.
60. "Nonlinear L2 anti-windup for enlarged stability regions and regional performance", S. Galeani, S. Onori, A.R. Teel, and L. Zaccarian, *Proceedings of the 7<sup>th</sup> IFAC Symposium on Nonlinear Control Systems*, August 2007, Pretoria, South Africa.
61. "Hybrid systems: stability and control", *Proceedings of the 26<sup>th</sup> Chinese Control Conference*, pp. 29-36, July 2007, Zhangjiajie Hunan, China.
62. "Complex hybrid systems: stability analysis for Omega limit sets", C. Cai, R. Goebel, S. Sanfelice, and A.R. Teel, *Proceedings of the 26<sup>th</sup> Chinese Control Conference*, pp. 766--769, July 2007, Zhangjiajie Hunan, China.
63. "Homogeneous hybrid systems and a converse Lyapunov theorem", S.E. Tuna and A.R. Teel, *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 6235 – 6240, San Diego, CA, December 2006.
64. "Summability criteria for stability of sets for sampled-data nonlinear inclusions", D. Nesic, A. Loria, E.V. Panteley, and A.R. Teel, *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 4265 – 4270, San Diego, CA, December 2006.
65. "On stability of sets for sampled-data nonlinear inclusions via their approximate discrete-time models", D. Nesic, A. Loria, E.V. Panteley, and A.R. Teel, *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 4253 – 4258, San Diego, CA, December 2006.
66. "On the robust stability and stabilization of sampled-data systems: a hybrid systems approach", P. Naghshtabrizi, J. Hespanha, and A.R. Teel, *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 4873 – 4878, San Diego, CA, December 2006.
67. "Simple robust control invariant tubes for some classes of nonlinear discrete time systems", S. Rakovic, A.R. Teel, D.Q. Mayne, A. Astolfi, *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 6397 – 6402, San Diego, CA, December 2006.
68. "On the continuity of asymptotically stable compact sets for simulations of hybrid systems", R. Sanfelice and A.R. Teel, *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 270 – 275, San Diego, CA, December 2006.



69. "Smooth patchy control Lyapunov functions", R. Goebel, C. Prieur and A.R. Teel, *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 3271 – 3276, San Diego, CA, December 2006.
70. "LMI-based linear anti-windup for discrete time linear control systems", M. Massimetti, L. Zaccarian, T. Hu, and A.R. Teel, *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 6173 – 6178, San Diego, CA, December 2006.
71. "Further results on static linear anti-windup design for control systems subject to magnitude and rate saturation", S. Galeani, S. Onori, A.R. Teel, and L. Zaccarian, *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 6373 – 6378, San Diego, CA, December 2006.
72. "Explicit Lyapunov functions for stability and performance characterizations for FOREs connected to an integrator", L. Zaccarian, D. Nesic, and A.R. Teel, *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 771 – 776, San Diego, CA, December 2006.
73. "Lyapunov analysis of sample-and-hold hybrid feedback", R. Sanfelice and A.R. Teel, *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 4879 – 4884, San Diego, CA, December 2006.
74. "Relaxation theorems for hybrid systems", C. Cai, A.R. Teel, and R. Goebel, *Proceedings of the 45<sup>th</sup> IEEE Conference on Decision and Control*, pp. 276 – 281, San Diego, CA, December 2006.

### Book Chapters

75. "Hybrid control systems", A.R. Teel, R.G. Sanfelice, and R. Goebel, In *Encyclopedia of Complexity and System Science*, Springer, 2009, to appear.
76. "Robust hybrid control systems: an overview of some recent results", A.R. Teel, *Advances in Control Theory and Applications*, C. Bonivento et al. (Eds), LNCIS 353, pp. 279-302, Springer-Verlag, 2007.
77. "Hybrid systems: limit sets and zero dynamics with a view toward output regulation", C. Cai, R. Goebel, R.G. Sanfelice, and A.R. Teel, In *Analysis and Design of Nonlinear Control Systems*, In Honor of Alberto Isidori, A. Astolfi and L. Marconi (Eds.), pp. 237—258, Springer-Verlag, 2007.
78. "Case studies on the control of input-constrained linear plants via output feedback containing an internal deadzone loop", D. Dai, T.S. Hu, and AR Teel, et al. In *Advanced Strategies in control systems with input and output constraints*, LNCIS 346, Springer, pp. 313-340, 2007.

79. "Nonlinear anti-windup for exponentially unstable linear plants", S. Galeani, A.R. Teel, and L. Zaccarian, In *Current Trends in Nonlinear Systems and Control*, Menini, Zaccarian and Abdallah, eds., pp. 143-162, Birkhauser, Boston, MA, 2006.
80. "Dual matrix inequalities in stability and performance analysis of linear differential/difference inclusions", R. Goebel, T. Hu, and A.R. Teel, In *Current Trends in Nonlinear Systems and Control*, Menini, Zaccarian and Abdallah, eds., pp. 103-122, Birkhauser, Boston, MA, 2006.
81. "On the literature's two different definitions of uniform global asymptotic stability for nonlinear systems", A.R. Teel and L. Zaccarian, In *Advanced Topics in Control Systems Theory, Lecture Notes in Control and Information Sciences*, 328, Loria, Lamnabhi-Lagarrigue, and Panteley, eds., pp. 285-289, Springer, London, 2006.



## Research Accomplishments

### Hybrid control design

*Global asymptotic stabilization of the attitude of an underactuated rigid body:* The problem of globally asymptotically stabilizing attitude for an underactuated rigid body is relevant for satellite pointing and tracking. The problem is a challenging one for multiple reasons. Locally, under actuation makes the linearized model non-controllable and so nonlinear effects must be exploited to achieve convergence to the desired orientation. In fact, to achieve robust local asymptotic stabilization, non-classical (hybrid) control algorithms are necessary. Globally, orientation problems present inherent topological constraints that preclude global stabilization using classical techniques. Fortunately, both local and global obstructions to stabilization can be overcome using hybrid feedback. We illustrated this fact in [41] where we developed nontrivial hybrid feedback control laws to achieve global asymptotic hybrid feedback stabilization for the orientation of an underactuated rigid body.

*Autonomous vehicle source seeking:* A problem of interest to the Air Force is the design of control algorithms to enable autonomous vehicles to locate the source of an intensity field, using only measurements of the strength of the intensity field. The vehicle's dynamic constraints play an important role in the efficient designs of such algorithms. In our work, we showed how to exploit efficient line search optimization algorithms in the context of source seeking via autonomous vehicles with nonholonomic constraints [36,39,47]. The backbone for understanding the overall dynamical behavior, which combines vehicle dynamics and the dynamics of the optimization algorithm, is hybrid systems theory. The algorithms can be designed so that the overall closed-loop system is "well-posed", in the sense that it is guaranteed to be robust to small perturbations and measurement noise.

*Control of systems with impacts (juggling control):* A largely unexplored area of control design arises when control can only be exerted through impacts on the mechanism to be controlled. A prototype for such a control problem involves stabilization a set of balls to a rhythmic pattern corresponding to juggling. This problem poses a wealth of challenges. We have made considerable headway by modeling control via impacts using our recent mathematical formulation of hybrid dynamical systems, which includes the notion of hybrid time domains. The characterization of tracking is nontrivial, since the actual system and the system to track may experience impacts at different times. Nevertheless, an efficient description of tracking can be given and can be achieved with feedback control [45].

*Supervisors of hybrid controllers for global asymptotic stabilization:* The concept of hybrid supervisors for classical controllers in order to improved performance through intelligent switching has been in the literature for over a decade. We extended this idea to the supervision of hybrid, rather than classical, controllers. This extension helps to make more modular the design of complex hybrid control systems. The ideas can be used in a wide variety of control problems including problems for which no classical

feedback controller exists. Our ideas on this topic, which generalize our earlier results on "throw-and-catch" control [51,38], hybrid patchy feedbacks [11], and patchy control Lyapunov functions [58], were documented in [32,75].

### Hybrid systems analysis

*Homogeneity, Zeno behavior, and "linearizations":* A fascinating phenomenon in hybrid dynamical systems is Zeno behavior, which occurs when a solution experiences an infinite number of jumps in a finite amount of time. Researchers have exerted much effort to characterize when hybrid systems will admit Zeno behavior. In our work, we gave necessary and sufficient Lyapunov conditions for a stability property that is strongly related to Zeno stability, and we fully characterized Zeno behavior for systems that are approximately homogeneous. In turn, this homogeneity study permitted us to define "linearizations" for hybrid systems and deduce local asymptotic stability properties from those of the linearization [34,37].

*Matrosov functions:* A commonly used tool for establishing asymptotic stability in classical and hybrid systems is the Lyapunov function. In the hybrid setting, a Lyapunov function is a positive definite function that decreases along flows and at jumps prescribed by the hybrid system's data. While smooth Lyapunov functions always exist for hybrid systems (see, for example, [14,4]), they can be difficult to find. With this in mind, additional analysis tools that relax the standard Lyapunov conditions are desirable and helpful for analysis. In this direction, invariance principles are certainly useful. See, for example, [10]. Another related tool comes through the notion of "Matrosov functions" which are based on an idea due to Matrosov in the 1960's for time-varying systems. Matrosov functions comprise a finite family of functions, none of which is always decreasing but for which the first one is not increasing and is decreasing at some places in the state space, the next is not increasing where the one before it is not decreasing, and so on in such a way that the entire state space is covered. The existence of Matrosov functions for a hybrid system guarantees asymptotic stability. We established such a result and illustrated its application in [40].

*Invariance principle applied to switching systems:* An often-studied class of systems is so-called "switching systems". These systems can be viewed as a special case of hybrid systems and then invariance principles for hybrid systems can be applied to them. To illustrate the power of the invariance principle we developed in [10], we applied it to switching systems in [3]. In this work, we showed that our results cover all known existing results on invariance for switching systems and we also provide significant extensions. These theorems allow us to establish asymptotic stability for switching systems under weaker conditions than have appeared in the literature previously.

*Analysis of Networked Control Systems:* Networked control systems comprise several systems controlled over a common communication network. Such systems are becoming more and more common in industrial and battlefield applications. Stability in such systems can be somewhat difficult to predict due to communication variability,

delays, and dropouts. Networked control systems can be modeled as hybrid dynamical systems and thus stability analysis tools like Lyapunov functions, Matrosov functions, and invariance principles can be applied to study stability. We continued to develop new Lyapunov-based and input-output-based analysis tools that can be used to certify asymptotically stable behavior in networked control systems [13,18,5]. Our results provide some of the least conservative results available in this area.

*Additional work:* In addition to our work on hybrid systems, we continued to make contributions in the areas of anti-windup synthesis [7,9,19,20,60], direct design for systems with actuator saturation [78,55], and model predictive control [12,50]. Our purpose here is to refine our previous results on these topics, preparing the way for the introduction of hybrid control aspects into these feedback control methodologies.

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